2007 AM2 EX−2 Locutus

1.

$$I R + L \frac{dI}{dt} = V(t)$$

$$L = 10^{-3}\ H,\ R = 20\ \Omega,\ V = 0.2\, e^{2t}$$

$$\Rightarrow I' + \frac{R}{L} I = \frac{V}{L},\ \text{Let}\ P = \frac{R}{L} = 2 * 10^4,\ Q = \frac{0.2}{L} = 2 * 10^2$$

$$\Rightarrow I' + P I = Q\, e^{2t}$$

$$I = e^{-\int P\, dt} \left(\int Q\, e^{2t}\, e^{\int p\, dt}\, dt + C \right) =$$

$$e^{-Pt} \left(\int Q\, e^{2t}\, e^{Pt}\, dt + C \right) = e^{-Pt} \left(\frac{Q}{P+2} e^{(P+2)t} + C \right) = \frac{Q}{P+2} e^{2t} + C\, e^{-Pt}$$

$$t = 0,\ I = 0 \Rightarrow C = \frac{-Q}{P+2} \approx -0.01$$

$$\therefore I = 0.01 \left(e^{2t} - e^{-2*10^4 t} \right)$$

2.

$$y'' + P(x)\, y' + Q(x)\, y = 0$$

$$W = C\, e^{-\int P\, dx} \Rightarrow W = c\, e^{-\int \frac{1}{x}\, dx} = \frac{c}{x},\ x \in (0, \infty)$$

3.

same as 2.

$$W = c\, e^0 = C,\ x \in (-\infty, \infty)$$

4.

$$\text{Let}\ u = e^{\lambda t}$$

$$\Rightarrow \lambda^2 + 4\lambda - 4 = 0 \Rightarrow \lambda = -2 \pm 2\sqrt{2}$$

$$\therefore u = e^{-2t} \left(C_1\, e^{2\sqrt{2}\, t} + C_2\, e^{-2\sqrt{2}\, t} \right)$$

5.

same as 4.

$$\lambda = \frac{-5 \pm \sqrt{13}}{2}$$

$$\therefore u = e^{\frac{-5}{2} t} \left(C_1\, e^{\frac{\sqrt{13}}{2} t} + C_2\, e^{\frac{-\sqrt{13}}{2} t} \right)$$

6.

$$u' - \frac{1}{x^2} u = 2\, \frac{1}{x^2}\, \text{Sin}\, \frac{1}{x}$$

$$u = e^{-\int P\,dx}\left(\int Q\,e^{\int P\,dx}\,dx + C\right)$$

$$\int P\,dx = \int \frac{-1}{x^2}\,dx = \frac{1}{x}$$

$$\therefore u = e^{\frac{-1}{x}}\left(\int 2\,\frac{1}{x^2}\,\sin\frac{1}{x}\,e^{\frac{1}{x}}\,dx + C\right)$$

$$\text{Define } A \equiv \int 2\,\frac{1}{x^2}\,\sin\frac{1}{x}\,e^{\frac{1}{x}}\,dx,\quad y \equiv \frac{1}{x}$$

$$dy = -y^2\,dx$$

$$A = 2\int y^2 \sin y\,e^y\left(\frac{-1}{y^2}\right)dy = -2\int \sin y\,e^y\,dy = -2\left(\sin y\,e^y - \int e^y \cos y\,dy\right) =$$

$$-2\left(\sin y\,e^y - e^y \cos y - \int e^y \sin y\,dy\right) = 2\,e^y\,(\cos y - \sin y) - A$$

$$\therefore A = e^y\,(\cos y - \sin y)$$

$$\Rightarrow u = C\,e^{\frac{-1}{x}} + \cos\frac{1}{x} - \sin\frac{1}{x}$$