# **Applied Math IV**: Example Sheet 2

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1. **Fourier transform:** Fourier transform is defined as $\widetilde{f}(k) = \int_{-\infty}^{\infty} f(x)\exp(ikx)dx$. Compute $\widetilde{f}(k)$ of the following functions, and draw both $f(x)$ and $\widetilde{f}(k)$:

   (a) a top-hat window: $f(x) = \begin{cases} 1, & \text{when } |x| < 1/2, \\ 1/2, & \text{when } |x| = 1/2, \\ 0, & \text{when } |x| > 1/2. \end{cases}$ (7%)

   (b) a triangular window: $f(x) = \begin{cases} 1-|x|, & \text{when } |x| \leq 1, \\ 0, & \text{when } |x| > 1. \end{cases}$ (7%)

   (c) a Gaussian window/function: $f(x) = \exp(-\pi x^2)$. (7%)

   (d) a Dirac Delta: $f(x) = \delta(x) = \begin{cases} \infty, & \text{when } x = 0, \\ 0, & \text{when } x \neq 0. \end{cases}$ (7%)

   (e) a sampling function: $f(x) = \sum_{n=-\infty}^{\infty} \delta(x-n)$. (7%)

2. **Convolution & Correlations:** Compute the following convolutions or correlations of the functions in question 1:

   (a) (1a) * (1a). (7%)

   (b) (1c) * (1c). (7%)

   (c) (1c) * (1d). (7%)

   (d) (1c) * (1e). (7%)

   (e) Corr((1c), (1c)). (7%)

   (f) Corr((1c), (1e)). (7%)

3. **Deconvolution:**

   (a) Given $g * h(x) = g(x) = (1c)$, what is $h(x)$? (7%)

   (b) Given $g * h(x) = (1b)$ and $g(x) = (1a)$, what is $h(x)$? (7%)

4. **Differential equations:** In a 3-dimensional (3D) space, given $\rho(\mathbf{x}) = -\nabla \cdot \mathbf{v}(\mathbf{x})$, where $\rho(\mathbf{x})$ is a scalar, $\mathbf{x} \equiv x\hat{x} + y\hat{y} + z\hat{z}$ is the 3D coordinates, and $\mathbf{v}(\mathbf{x}) \equiv v_x\hat{x} + v_y\hat{y} + v_z\hat{z}$ is a 3D vector field. If we already know $\widetilde{\rho}(\mathbf{k})$ where $\mathbf{k} \equiv k_x\hat{k_x} + k_y\hat{k_y} + k_z\hat{k_z}$, find at least one solution for $\widetilde{\mathbf{v}}(\mathbf{k})$, i.e. derive $\widetilde{v}_x(\mathbf{k})$, $\widetilde{v}_y(\mathbf{k})$, $\widetilde{v}_z(\mathbf{k})$, all as functions of $\widetilde{\rho}(\mathbf{k})$. (9%)