Applied Math IV: Example Sheet 4

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1. **Definitions:** What are the orders and degrees of the following equations? Are they linear or non-linear, homogeneous or inhomogeneous? In addition, for (d) and (e), are they elliptic, parabolic, or hyperbolic?

   (a) $\frac{\partial^2 u}{\partial x^2} + u = 0$.

   (b) $(x+1)\frac{\partial^2 u}{\partial x^2} = x^4$.

   (c) $(u+1)\frac{\partial^2 u}{\partial x^2} = u^4$.

   (d) The 2D Laplace's equation: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

   (e) $(x^2 - y^2 - 1)\frac{\partial^2 u}{\partial x^2} + 2x\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y^2 = 0$.

2. **2nd-order linear PDE:** Consider the following equation:

$$(x-1)\frac{\partial^2 u}{\partial x^2} + \sqrt{y}\frac{\partial^2 u}{\partial x \partial y} + (x+1)\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} = x. \tag{1}$$

   (a) For a differential region near the origin $x = y = 0$, is this PDE elliptic, parabolic, or hyperbolic?

   (b) Draw an x-y diagram to show the regions whitin which the PDE is elliptic, parabolic, or hyperbolic.

3. **Wave equation:** A tightly stretched string, with its ends fixed at the points $(0, 0)$ and $(bL, 0)$ (b is a constant), hangs at rest under its own weight. The y axis points vertically upward.

   (a) Find the describing equation for the position $u(x)$ of the string.

   (b) If $u(x) = g[(x/2 - L)^2 - L^2]/(2a^2)$ is a solution (where $a^2 = P/m$), what is the value of b?

   (c) If the string is vibrating and subject to both gravity loading and viscous drag, what is the describing equation for $u(x)$? (Suppose the strength of the viscous drag per unit length of string can be expressed as $c|\partial u/\partial t|$, $c > 0$.)

4. **Diffusion equation:** Consider an elemental slice of length Δx of a long, slender rod.

   (a) If the rod is insulated, what is the heat equation (the equation of T)?

   (b) If there is heat generation of rate $\phi(x, t)$ within the rod, what is the heat equation?

   (c) If the rod is not insulated, then the convection in the surrounding fluid will induce an extra heat transfer. The rate of this heat loss is given by $Q = hA(T - T_f)$, where h is the convection coefficient, A is the surface area of the rod, and T_f is the temperature of the surrounding fluid. What is the heat equation now, including the effect in (b)?