Applied Math IV: Example Sheet 1

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1. **Definitions:** What are the orders and degrees of the following equations? Are they linear or non-linear, homogeneous or inhomogeneous? In addition, for (d) and (e), are they elliptic, parabolic, or hyperbolic?

   (a) $\frac{\partial^2 u}{\partial x^2} = u$.

   (b) $(x+1)\frac{\partial^3 u}{\partial x^3} + x^2 = 0$.

   (c) $(u+1)\frac{\partial^3 u}{\partial x^3} + u^2 = 0$.

   (d) The 2D Laplace's equation: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

   (e) $(x^2 - y^2 - 1)\frac{\partial^2 u}{\partial x^2} + 2x\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y^2 = 0$.

2. **Linear dependence:** Consider the following two sets of functions. Are the functions in the same set linearly independent?

   (a) $\sinh x$, $\exp(x) - \exp(-x)$, 1.

   (b) $\cos 2x - 1$, $\sin^2 x$, 1.

3. **$\mathbf{2^{nd}}$-order linear PDE:** Consider the following equation:

$$(x-1)\frac{\partial^2 u}{\partial x^2} + \sqrt{y}\frac{\partial^2 u}{\partial x \partial y} + (x+1)\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} = x. \tag{1}$$

   (a) For a differential region near the origin $x = y = 0$, is this PDE elliptic, parabolic, or hyperbolic?

   (b) Draw an x-y diagram to show the regions whitin which the PDE is elliptic, parabolic, or hyperbolic.

4. **Diffusion:** The axis of symmetry of a horn coincide with the z-axis in an (x, y, z) coordinate system. The surface mass density of the horn is ρ (constant). The body of the horn can be described by $x^2 + y^2 = 1 + z$ $(0 < z < 10)$. The temperature $T(z, t)$ is a function of z and t. The thermal conductivity K and the thermal capacity C are both constant. Please derive the differential equation that $T(z, t)$ satisfies.