Cosmology: Final Exam

Jiun-Huei Proty Wu 3:30pm-8:30pm, June 21, 2007

Instruction: Answer all four questions. The time limit is five hours. **NO** books, notes, or references are allowed in the exam. (作答時請盡量用中文回答,以讓他人看得懂為原則.不可使用任何書籍或字典,題目中的英文若看不懂可以發問)

Equations that you may use:

$$H^2+\frac{k}{a^2}=\frac{8\pi G\rho}{3}+\frac{\Lambda}{3},\quad \frac{\ddot{a}}{a}=\frac{-4\pi G(\rho+3P)}{3}+\frac{\Lambda}{3},\quad H=\frac{1.66}{m_{Pl}}N^{1/2}T^2,$$

$$s=\frac{2\pi^2}{45}N_sT^3 \text{ where } N_s=\sum_b g_i\left(T_i/T\right)^3+\frac{7}{8}\sum_f g_i\left(T_i/T\right)^3,$$

$$n_i\approx 1.2g_iT_i^3/\pi^2,\ \rho_i=g_iT_i^4\pi^2/30 \quad \text{(relativistic Bosons)},$$

$$n_i\approx 3.6g_iT_i^3/(4\pi^2),\ \rho_i=7g_iT_i^4\pi^2/240 \quad \text{(relativistic Fermions)},$$

$$n_i=g_i[m_iT_i/(2\pi)]^{3/2}\exp(-m_i/T_i),\ \rho_i=m_in_i \quad \text{(non-relativistic particles)},$$

$$\ddot{\phi}+3H\dot{\phi}=-V'(\phi),\quad \varepsilon_V=\frac{1}{16\pi G}\left(\frac{V'}{V}\right)^2,\quad \eta_V=\frac{1}{8\pi G}\left|\frac{V''}{V}\right|,\quad N=-8\pi G\int_{\phi_i}^{\phi_f}\left(\frac{V}{V'}\right)d\phi$$

Constants you may use:

$$G=6.67\times10^{-8}\text{cm}^3\text{g}^{-1}\text{s}^{-2},\ c=3\times10^{10}\text{cm s}^{-1},\ 1\text{ Mpc}=3.1\times10^{24}\text{cm},\ T_0=2.73\text{K},$$

$$H_0=100h\text{ km s}^{-1}\text{ Mpc}^{-1}=\left(9.78\times10^9h^{-1}\text{yr}\right)^{-1},\ G_F=1.17\times10^{-5}\text{GeV}^{-2},$$

$$1\text{ GeV}=1.78\times10^{-24}g=\left(1.97\times10^{-14}cm\right)^{-1}=1.16\times10^{13}K.$$

1. Answer the following questions:
   (a) Is **inflation** happening now and why? (5%)
   (b) If the universe is **closed**, then the first peak in the CMB power spectrum should move from $\ell\approx200$ to a lower or higher ℓ? Explain why. (5%)
   (c) If the half lifetime of neutrons were 100 minutes, then the **abundance of** ^{4}He today would be higher or lower? Explain why. (5%)
   (d) Among the components in the CMB anisotropy, write down **two super-degree** effects. (5%)
   (e) In April 2007, AMiBA detected the galaxy cluster A2142. Via which effect in the CMB did **AMiBA** detect the cluster? Explain the effect. (5%)
   (f) What are the **mechanisms** to form the cosmic perturbations that result in the CMB anisotropy and the structures of the universe today? (5%)

2. Consider a Chaotic inflation $V(\phi)=\frac{1}{2}m^2\phi^2$. Assume that the field rolls towards $\phi=0$ from the positive side, and that inflation starts at ϕ_i and ends when $\varepsilon_V=1$ or $\eta_V=1$, whichever comes first.
   (a) Calculate the field ϕ_f at the end of inflation. Give your answer in m_{Pl}. (5%)
   (b) If the inflation between ϕ_i and ϕ_f is just enough to resolve the cosmological conundrums such as the horizon and the flatness problems, what is ϕ_i? Give your answer in m_{Pl}. (5%)
   (c) Given that $\phi=\phi_i$ and $a=a_i$ at $t=0$, use PSRA to find the solutions $\phi(t)$ and $a(t)$. (10%)
   (d) If we use equation of state $P=w\rho$ to describe the inflaton, calculate w at ϕ_i and ϕ_f respectively. (5%)

3. Suppose a neutral and stable relativistic boson 'A' has a cross section $\sigma\approx 10^{-9}G_F^2T^2$. It interacts with particles that have effective number density $n=T^3$. Suppose A decouples from the thermal-equilibrium system at temperature T_d. For $T\geq T_d$, all particles in the universe are in thermal equilibrium and have an effective $N_s=200$. For $T_d>T>10\text{MeV}$, most particles start to annihilate so that when $10\text{MeV}>T>1\text{MeV}$ the only relativistic particles are A ($g_A=30$), photons ($g_\gamma=2$), neutrinos ($g_\nu=6$), and e^-e^+ ($g_{e^-e^+}=4$). Then the neutrinos decouple at 1 MeV, and e^-e^+ annihilate at 0.5 MeV. Particle A is still relativistic today.
   (a) Compute T_d (in GeV). (5%)
   (b) Given $T_\gamma=2.73\text{K}$ today, what is the temperature of particle A today? (10%)
   (c) What is $\bar{N}_A=n_A/s_{\text{tot}}$ today? Here n_A is the number density of particle A and s_{tot} is the total entropy density in the universe. (5%)
   (d) What is Ω_A today? (5%)

4. 請寫下你對這門課的感想、或改進建議、或是你學習上的困難、或是考試考不好的原因（用中文作答，5 字一分，滿分 20%）